Find $\binom{42}{7}$.

SCORE: _____ / 4 PTS

$$\frac{42!}{7!\,35!} = \frac{\cancel{42}\times 41\times \cancel{40}\times 39\times 38\times 37\times \cancel{36}^{12}}{\cancel{7}\times \cancel{6}\times \cancel{5}\times \cancel{4}\times \cancel{3}\times \cancel{2}\times 1}$$ 1 1

$$= 41\times 39\times 38\times 37\times 12$$ 1 ← ANY PRODUCT WHICH GIVES CORRECT FINAL ANSWER + DOESN'T INVOLVE DIVISION OR !

$$= 26{,}978{,}328$$ 1

Find the value of $\sum_{p=1}^{\infty} \frac{16}{3^{2p}}$. **HINT: Write out several terms of the series first.**

SCORE: _____ / 4 PTS

$$= \frac{16}{3^2} + \frac{16}{3^4} + \frac{16}{3^6} + \cdots$$

$\times \frac{1}{9}$ $\times \frac{1}{9}$

$$= \frac{\boxed{\frac{16}{9}}}{\boxed{1 - \frac{1}{9}}} = \frac{\frac{16}{9}}{\frac{8}{9}} = \boxed{2}$$

1 2 1

An employer ran a year-long charity campaign. During the seventh week, $716 was donated. **SCORE: _____ / 5 PTS**

[a] If the amount donated every week was $24 higher than the amount donated the previous week, how much was donated over the entire year (52 weeks) ?

$a_7 = a_1 + 6d$

$716 = a_1 + 6(24)$ ①

$a_1 = 572$ ½

$S_{52} = \frac{52}{2}(2(572) + 51(24))$ ①

$= 61,568$ ½

[b] If the amount donated every week was 3.2% higher than the amount donated the previous week, how much was donated the first week ? **Round your answer to the nearest cent.**

$a_7 = a_1 r^6$

$716 = a_1(1.032)^6$ ①

$a_1 = 592.70$ ①

Use the entries of Pascal's Triangle to expand and simplify $(7n^3 - 2n^5)^4$. 1 4 6 4 1 **SCORE: _____ / 5 PTS**

<u>You must show the intermediate step in the expansion to get full credit.</u>

$$\underbrace{(7n^3)^4} + \underbrace{4(7n^3)^3(-2n^5)} + \underbrace{6(7n^3)^2(-2n^5)^2} + \underbrace{4(7n^3)(-2n^5)^3} + \underbrace{(-2n^5)^4}$$

$$= \underbrace{2401n^{12}} \underbrace{- 2744n^{14}} \underbrace{+ 1176n^{16}} \underbrace{- 224n^{18}} \underbrace{+ 16n^{20}}$$

$\frac{1}{2}$ POINT EACH

Using mathematical induction, prove that $1^2 - 2^2 + 3^2 - 4^2 + \cdots + (-1)^{n+1} n^2 = (-1)^{n+1} \frac{n(n+1)}{2}$ **SCORE: _____ / 12 PTS**

for all positive integers n.

BASIS STEP : $1^2 = 1 = (-1)^2 \frac{1(2)}{2}$

INDUCTIVE STEP:

ASSUME $1^2 - 2^2 + 3^2 - 4^2 + \cdots + (-1)^{k+1} k^2 = (-1)^{k+1} \frac{k(k+1)}{2}$

FOR SOME PARTICULAR BUT ARBITRARY INTEGER $k \geq 1$

$1^2 - 2^2 + 3^2 - 4^2 + \cdots + (-1)^{k+2}(k+1)^2$

$= 1^2 - 2^2 + 3^2 - 4^2 + \cdots + (-1)^{k+1} k^2 + (-1)^{k+2}(k+1)^2$

$= (-1)^{k+1} \frac{k(k+1)}{2} + (-1)^{k+2}(k+1)^2$

$= (-1)^{k+1}(k+1)\left[\frac{k}{2} + (-1)(k+1)\right]$

$= (-1)^{k+1}(k+1)\left(\frac{k}{2} - k - 1\right)$

$= (-1)^{k+1} \frac{k+1}{2}(k - 2k - 2)$

$= (-1)^{k+1} \frac{k+1}{2}(-k-2)$

$= (-1)^{k+1} \frac{k+1}{2}\left[-(k+2)\right]$

$= (-1)^{k+2} \frac{(k+1)(k+2)}{2}$

BY M.I.

$1^2 - 2^2 + 3^2 - \cdots + (-1)^{n+1} n^2$

$= (-1)^{n+1} \frac{n(n+1)}{2}$

FOR ALL POSITIVE INTEGERS